

# 역변환과 도함수의 변환

부분분수, 표의 역변환, 초기값 문제를 대수방정식으로

공학수학 I · 강의노트 §4.2 Example 풀이

## 1 역변환

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} &= \frac{t^2}{2}, \\ \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} &= t - \frac{48}{4!}t^4 = t - 2t^4, \\ \mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} &= \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{3}{s^2} + \frac{3}{s^3} + \frac{1}{s^4}\right\} = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3, \\ \mathcal{L}^{-1}\left\{\frac{1}{4s^2+1}\right\} &= \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1/4}\right\} = \frac{1}{2}\sin\frac{t}{2}, \\ \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\} &= \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t.\end{aligned}$$

## 2 $\mathcal{L}^{-1}\left\{\frac{-2s+6}{s^2+4}\right\}$

$$\frac{-2s+6}{s^2+4} = -2\frac{s}{s^2+4} + 3\frac{2}{s^2+4} \Rightarrow \mathcal{L}^{-1} = -2\cos 2t + 3\sin 2t.$$

## 3 $y' + 3y = 13\sin 2t, y(0) = 6$

$$(s+3)Y - 6 = \frac{26}{s^2+4} \Rightarrow Y = \frac{6}{s+3} + \frac{26}{(s+3)(s^2+4)}.$$

$\frac{26}{(s+3)(s^2+4)} = \frac{2}{s+3} + \frac{-2s+6}{s^2+4}$  이므로 (앞 절의 역변환을 그대로 쓴다)

답

$$y = 8e^{-3t} - 2\cos 2t + 3\sin 2t$$

## 4 초기값 문제

(1)  $y'' + y = \sqrt{2}\sin\sqrt{2}t, y(0) = 10, y'(0) = 0.$

$$s^2Y - 10s + Y = \frac{2}{s^2+2} \Rightarrow Y = \frac{10s}{s^2+1} + \frac{2}{(s^2+1)(s^2+2)} = \frac{10s}{s^2+1} + \frac{2}{s^2+1} - \frac{2}{s^2+2}.$$

답

$$y = 10\cos t + 2\sin t - \sqrt{2}\sin\sqrt{2}t$$

(2)  $2y''' + 3y'' - 3y' - 2y = e^{-t}, y(0) = y'(0) = 0, y''(0) = 1.$  변환하면  $2(s^3Y - 1) + 3s^2Y - 3sY - 2Y = \frac{1}{s+1},$   
즉

$$(2s^3 + 3s^2 - 3s - 2)Y = 2 + \frac{1}{s+1} = \frac{2s+3}{s+1}.$$

$2s^3 + 3s^2 - 3s - 2 = (s - 1)(2s + 1)(s + 2)$  이므로

$$Y = \frac{2s + 3}{(s + 1)(s - 1)(2s + 1)(s + 2)} = \frac{1/2}{s + 1} + \frac{5/18}{s - 1} - \frac{8/9}{s + 1/2} + \frac{1/9}{s + 2}.$$

답

$$y = \frac{1}{2}e^{-t} + \frac{5}{18}e^t - \frac{8}{9}e^{-t/2} + \frac{1}{9}e^{-2t}$$

확인:  $y(0) = \frac{1}{2} + \frac{5}{18} - \frac{8}{9} + \frac{1}{9} = 0.$